CHARACTERIZATION OF METAMATERIAL SLABS IN TERMS OF SCATTERING PARAMETERS UNDER OBLIQUE PLANE WAVE INCIDENCE

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ABSTRACT

In this work, a retrieval procedure which allows the determination of equivalent dyadic permittivity and permeability of metamaterials from reflection and transmission coefficients obtained for several incidence directions and polarizations is presented. The main goal is to determine to which extent a model of anisotropic homogeneous material can be applied to specific metamaterials. Preliminary results on wire media and arrays of magnetic resonators show that a certain level of symmetry in the unit cell is required to avoid bianisotropy, and that small unit cells compared to the wavelength are favorable for such a homogenization.

1. INTRODUCTION

Metamaterials (MTM) are broadly defined as artificial materials with unusual electromagnetic properties. In the present work, we are interested in MTM which consist of periodic structures with small lattice constant compared to the wavelength (a typical value being one tenth of the wavelength). For this kind of structures, it is often desirable to consider them as homogeneous media, characterized by equivalent medium parameters like the permittivity ε, the permeability μ or the refractive index n. There has recently been growing interest in MTM that can exhibit negative values of one or several of these parameters [1, 2].

Among the different homogenization procedures used to retrieve the equivalent medium parameters, one of the most common in the field of MTM consists in the determination of these parameters from reflection and transmission coefficients of a slab under plane wave incidence (scattering parameters) [3, 4]. To our knowledge, this approach has only been developed for normal incidence, and thus it only provides the equivalent medium parameters associated to this specific direction of propagation. In this work, this technique has been extended to oblique incidences, in order to investigate the properties of 2D or 3D MTM for different propagation directions, in particular to evaluate their isotropy. For that purpose, an improved retrieval procedure which allows extracting dyadic permittivity and permeability from reflection and transmission coefficients obtained for different incidences (oblique and normal) has been developed (Section 3) and tested on some MTM, like a wire medium or arrays of magnetic resonators (Section 4).

2. ANALYSIS OF METAMATERIAL SLABS

This section briefly describes how a metamaterial slab is characterized in terms of reflection and transmission coefficients under plane wave incidence. The method is similar to that used for the analysis of frequency selective surfaces (FSS) [5, 6].

2.1. Description

We consider here metamaterials as periodic structures with infinite periodicity in two directions (x and y) and finite periodicity in the third direction (z), as illustrated in Fig. 1. The structure shown in Fig. 1 is made of only one cell in the z direction, but in general there can be any finite number of cells in that direction. The lattice is rectangular, with lattice constants a and b in the x and y directions, respectively. The excitation is a plane wave with arbitrary incidence given by the angles θ and ϕ.

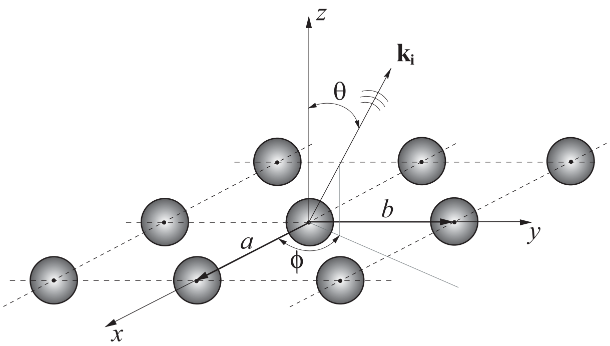

Figure 1. Metamaterial slab as a 2D infinite periodic structure arranged in a rectangular lattice and excited with an incident plane wave (wave vector $\mathbf{k}_i$).

The electromagnetic field near the structure (above and below) must satisfy the periodicity requirements imposed by Floquet's theorem. As a consequence, these fields can be expanded in a set of TM_{mn} and TE_{mn} modes (with respect to z), which are called Floquet modes [5, 6]. It can be shown that all these modes are plane waves with different wave vectors.

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An arbitrarily polarized incident plane wave can be expressed as a combination of the two fundamental Floquet modes, namely TE_{00} and TM_{00}. If the periodicity is small enough ($\max(a,b) < \lambda/2$), higher order Floquet modes are evanescent (in the *z* direction), and only the two fundamental modes propagate away from the structure. In other words, the reflected and transmitted scattered fields at a certain distance from the structure are essentially a combination of these two fundamental modes, and are thus plane waves with arbitrary polarization and wave vectors directed in a similar way as with an homogeneous slab (see Fig. 2). It can be noted that for normal incidence, the TE_{00} and TM_{00} become two TEM modes. This case can be considered as a limit case for oblique incidence where $\theta \to 0$.

2.2. Reflection and transmission coefficients

The performances of the metamaterial slab is characterized by reflection (ρ) and transmission (τ) coefficients associated with the two fundamental modes (TE_{00} and TM_{00}). Taking into account the interactions that may occur between these modes, the coefficients are defined by the following relations:

$$\begin{pmatrix} b_{\text{TM}}^{\text{refl}} \\ b_{\text{TE}}^{\text{refl}} \end{pmatrix} = \begin{pmatrix} \rho_{\text{TM-TM}} & \rho_{\text{TM-TE}} \\ \rho_{\text{TE-TM}} & \rho_{\text{TE-TE}} \end{pmatrix} \begin{pmatrix} a_{\text{TM}}^{\text{inc}} \\ a_{\text{TE}}^{\text{inc}} \end{pmatrix}$$
$$\begin{pmatrix} b_{\text{TM}}^{\text{trans}} \\ b_{\text{TE}}^{\text{trans}} \end{pmatrix} = \begin{pmatrix} \tau_{\text{TM-TM}} & \tau_{\text{TM-TE}} \\ \tau_{\text{TE-TM}} & \tau_{\text{TE-TE}} \end{pmatrix} \begin{pmatrix} a_{\text{TM}}^{\text{inc}} \\ a_{\text{TE}}^{\text{inc}} \end{pmatrix} \quad (1)$$

where the a^{inc}, b^{refl} and b^{trans} are the complex amplitudes of the incident, reflected and transmitted waves, respectively. $\rho_{\text{TM-TM}}$, $\rho_{\text{TE-TE}}$, $\tau_{\text{TM-TM}}$ and $\tau_{\text{TE-TE}}$ will be referred to as *co-polar* reflection and transmission coefficients, whereas $\rho_{\text{TM-TE}}$, $\rho_{\text{TE-TM}}$, $\tau_{\text{TM-TE}}$ and $\tau_{\text{TE-TM}}$ represent *cross-polar* coefficients.

2.3. Implementation with Ansoft HFSS

The procedure to compute the reflection and transmission coefficients defined in Eq. (1) with Ansoft HFSS is summarized as follows:

- The analysis of the 2D infinite periodic structure is reduced to the investigation of the unit cell with appropriate periodic boundary conditions (PBC). The phase shifts between the corresponding PBC are imposed by the direction of incidence.
- The structure is terminated in the *z* direction with absorbing boundary conditions, like PML (Perfectly Match Layers).
- The scattered field is calculated under plane wave incidence, which corresponds either to the TM_{00} or TE_{00} mode, with the chosen incidence direction.
- The reflection and transmission coefficients are calculated (using the HFSS *Fields Calculator*) by performing appropriate projections of the scattered electric field on the concerned mode function. These projections are made in transverse cross-sections (plane z = constant).

3. DYADIC MEDIUM PARAMETERS RETRIEVAL PROCEDURE

In this section, an improved retrieval procedure which allows obtaining the dyadic permittivity and permeability of an anisotropic slab from a set of reflection and transmission coefficients obtained for different incidence directions and polarizations is presented.

3.1. Hypotheses on the homogeneous material

Interactions between matter and electromagnetic fields are described by constitutive relations. If we assume that these interactions can be essentially represented by dipole moments, the material can be described by second order tensors (higher order multipoles are neglected). In this case, the constitutive relations take the general form of a bianosotropic medium:

$$\begin{cases} \mathbf{D} = \overline{\overline{\varepsilon}} \cdot \mathbf{E} + \overline{\overline{\xi}} \cdot \mathbf{H} \\ \mathbf{B} = \overline{\overline{\eta}} \cdot \mathbf{E} + \overline{\overline{\mu}} \cdot \mathbf{H} \end{cases} \quad (2)$$

The tensors $\overline{\overline{\xi}}$ and $\overline{\overline{\eta}}$ represent magneto-electric coupling. The material is thus characterized by 36 parameters (which are not all independent if we consider the reciprocity condition). However, developing a retrieval procedure allowing obtaining these 36 parameters from reflection and transmission coefficients is by no means an easy task. Some hypotheses must therefore be made on the material:

1) Magneto-electric coupling will not be considered ($\overline{\overline{\xi}} = \overline{\overline{\eta}} = 0$).
2) The Cartesian coordinate system is chosen such that $\overline{\overline{\varepsilon}}$ and $\overline{\overline{\mu}}$ can be represented by diagonal matrices:

$$\overline{\overline{\varepsilon}} = \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}, \quad \overline{\overline{\mu}} = \begin{pmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{pmatrix} \quad (3)$$

3) The material does not exhibit spatial dispersion.

The choice of material parameters considered here might appear somewhat limitative, in particular, neglecting the magneto-electric coupling, which can play an important role in MTM. However, the extent to which our retrieval procedure can be applied to a specific structure provides further insights on the magneto-electric coupling that occurs in that system. For example, a poor retrieval may indicate that not enough parameters have been taken into account in the chosen model to accurately describe the physical system under study. Hence, the key is not merely an exact retrieval procedure, but a procedure which exactitude can be evaluated.

3.2. Description of the slab problem

We consider a slab of homogeneous material described by Eq. (3) under oblique plane wave incidence, as shown in Fig. 2.

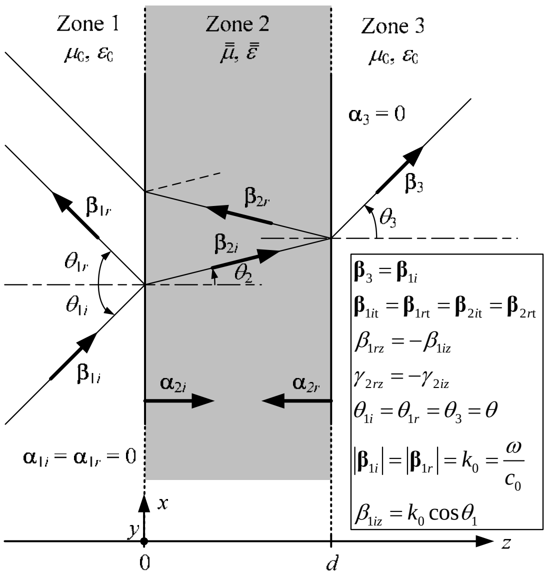


Figure 2. Oblique plane wave incidence on a planar anisotropic slab of thickness d. The frame on the right contains some relations on the propagation vectors that can be deduced from the continuity of the tangential fields at the two interfaces.

Zone 1 supports the incident (subscript '$_{1i}$') and reflected (subscript '$_{1r}$') waves. Zone 2 also supports two waves which are also referred to as incident (subscript '$_{2i}$') and reflected (subscript '$_{2r}$'). Zone 3 only supports the transmitted wave (subscript '$_{3}$'). The electric and magnetic fields of each plane wave have the general form:

$$\begin{cases} \mathbf{E}(\mathbf{r}) = \mathbf{E}_0\, e^{-\boldsymbol{\gamma}\cdot\mathbf{r}} \\ \mathbf{H}(\mathbf{r}) = \mathbf{H}_0\, e^{-\boldsymbol{\gamma}\cdot\mathbf{r}} \end{cases} \tag{4}$$

where $\boldsymbol{\gamma}$ is the (complex) propagation vector, which can be decomposed in its real and imaginary parts, or in its tangential and normal components (to the interface):

$$\boldsymbol{\gamma} = \boldsymbol{\alpha} + j\boldsymbol{\beta} = \boldsymbol{\gamma}_t + \gamma_z \mathbf{z} \tag{5}$$

3.3. Choice of incidence directions and polarizations

We consider incidence directions with ϕ = 0° or 90°, with TM or TE polarization (the angle ϕ determines the incidence plane, as in Fig. 1). In these cases, for an anisotropic slab described by Eq. (3), the reflected and transmitted waves keep the same polarization as the incident wave, thus avoiding "cross-polarization effects". This means that the cross-polar coefficients (as defined in section 2.2) will be zero, thus simplifying the retrieval procedure. The dispersion relation for a plane wave propagating in the anisotropic slab for the four considered cases is shown in Tab. 1 (reported at the end of the paper).

3.4. Reflection and transmission coefficients

The (co-polar) reflection and transmission coefficients ρ and τ are defined in terms of ratios of the tangential (to the interfaces) electric field, as expressed in Eq. (6), where the subscript '$_t$' holds for '$_x$' or '$_y$', depending on the incidence direction and polarization.

$$\begin{cases} \rho = \dfrac{E_{1rt}(x,y,0)}{E_{1it}(x,y,0)} = \dfrac{E_{01rt}}{E_{01it}} \\ \tau = \dfrac{E_{3t}(x,y,d)}{E_{1it}(x,y,0)} = \dfrac{E_{03t}}{E_{01it}}\, e^{-j\beta_{1iz}d} \end{cases} \tag{6}$$

This definition is consistent with the definition of the co-polar coefficients in Eq. (1). For the chosen incidence directions and polarizations, it can be shown that the reflection and transmission coefficients are given by:

$$\begin{cases} \rho = \dfrac{\rho_0(1-u^2)}{1-u^2\rho_0^2} \\ \tau = \dfrac{u(1-\rho_0^2)}{1-u^2\rho_0^2} \end{cases} \quad \text{with} \quad \begin{cases} u = e^{-\gamma_{2iz}d} \\ \rho_0 = \dfrac{\tilde{z}-1}{\tilde{z}+1} \end{cases} \tag{7}$$

where $\tilde{z}$ is a modified normalized impedance which is defined differently in each case (see Tab. 1). We have also defined a modified refractive index $\tilde{n}$:

$$\tilde{n} = \frac{\gamma_{2iz}}{j\beta_{1iz}} = \frac{\gamma_{2iz}}{jk_0\cos\theta} \tag{8}$$

3.5. Extraction of dyadic medium parameters

In fact, the modified parameters $\tilde{z}$ and $\tilde{n}$ have been defined in such a way that the expressions of ρ and τ given in Eq. (7) have the same form as for normal incidence with an isotropic medium. We can therefore use the same approach to extract $\tilde{z}$ and $\tilde{n}$ from ρ and τ

as described in [4], which is why it is not recalled here. Since $\tilde{z}$ and $\tilde{n}$ are known, information on the medium parameters can be deduced from the dispersion relation, the definition of $\tilde{z}$ (see Tab.1) and the definition of $\tilde{n}$ (Eq. (8)). It is found that for each of the four cases defined in section 3.3 (see Tab. 1):

- Three of the six medium parameters are involved.
- One of them can be determined directly by considering only one incidence direction (one value of θ).
- A relation between the two remaining parameters is obtained, which depends on θ.

Therefore, it is possible to determine the two remaining parameters by considering two different incidence directions (two values of θ), and by solving a system of two equations and two unknowns. This has a sense only if the two involved parameters do not depend on θ, i.e. if there is no spatial dispersion on these parameters.

At this stage, the question of the number of different incidence directions and polarizations that should be considered arises. By comparison with the retrieval procedure for normal incidence on isotropic slab [3, 4], where two equations with two unknowns are obtained (ρ, τ = *function*(ε,μ)), the present method offers a redundancy of information. Indeed, we can consider the four cases of Tab. 1 (ϕ = 0° or 90°, TM or TE), with different values of θ. In principle, we should always find the same parameters. If it is not the case, this means that the structure under study cannot be characterized by an homogeneous anisotropic medium described by Eq. (3), and that at least one of the assumptions made in section 3.1 is not valid. These aspects are investigated in more details on some specific MTM in section 4.

4. APPLICATION TO SPECIFIC MTM

In this section, some specific MTM are investigated using the approach presented in sections 2 and 3. The goal is essentially to evaluate the accuracy of the homogeneous anisotropic model described by Eq. (3) for the considered MTM.

For each of the considered structures, the analysis has been performed only with one cell in the *z* direction. The structures only contain perfect electric conductors (PEC).

4.1. The 1D wire medium

The wire medium is a MTM which exhibits plasma-like permittivity when the electric field is directed along the wires [1]. The unit cell of the considered structure is shown in Fig. 3. The dyadic permittivity and permeability have been extracted with the technique presented in Section 3. We have considered the four cases listed in Tab. 1, with θ = 0° and 40°. With the redundancy of information provided by this technique, we obtain in fact two instances of each parameters, which are shown in Fig. 4.

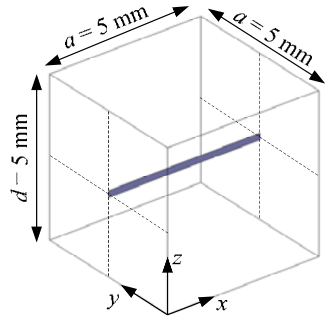


Figure 3. Unit cell of the 1D wire medium. The wires are infinitely thin vertical metallic strips (PEC) of width w = 0.2 mm. The wires are infinite in the x direction. We consider that the thickness of the equivalent homogeneous slab is d = 5 mm.

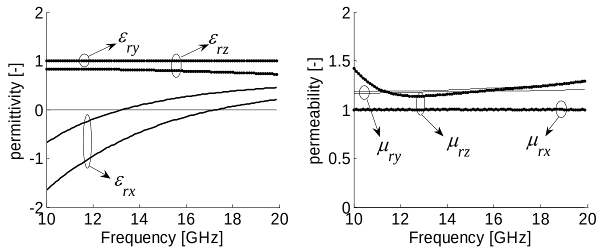


Figure 4. Equivalent dyadic permittivity and permeability for the 1D wire medium (real part only).

As expected, ε_x exhibits a plasma-like behavior whereas all the other parameters remain close to unity. It can also be observed that the two instances of ε_y and μ_x are exactly the same, whereas it is not the case for the other parameters. The fact that ε_x is very different between θ = 0° and 40° (pol. TM, ϕ = 0°) is a well known manifestation of spatial dispersion [7], but no explanation is found concerning the differences for the other parameters. Nevertheless, it can be noted that at the plasma frequency of 13.1 GHz (frequency where ε_x = 0), the unit cell is not so small compared to the wavelength (λ/a = 4.6), which limits the validity domain of homogenization procedures.

4.2. The loaded 1D wire medium

In order to lower the plasma frequency, and thus the size of the unit cell compared to the wavelength in the frequency range of interest, the wires are loaded with lumped inductances of 50 nH (one per unit cell). The extracted parameters for the same cases as for the unloaded wire medium are shown in Fig. 5.

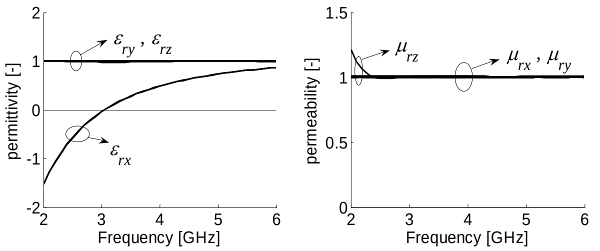


Figure 5. Equivalent dyadic permittivity and permeability for the 1D inductively loaded wire medium (real part only).

It can be seen that the different instances of the same parameters are identical (except for a small difference of μ_z at low frequencies), which means that this structure can be accurately described by an equivalent anisotropic homogeneous medium. For this structure, at the plasma frequency of 3 GHz, the unit cell is quite small compared to the wavelength (λ/a = 20), which is favourable for homogenization. It can also be noted that no spatial dispersion on ε_x is observed for the loaded wire medium.

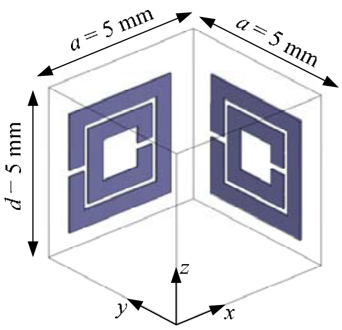
4.3. 2D array of split-ring resonators (SRR)

The scattering parameters of the two-dimensional (2D) array of planar split-ring resonators (SRR) [2] shown in Fig. 6 (a) have been computed for nine incidence directions (ϕ = 0°, 45°, 90° with θ = 0°, 30°, 60°), for both TM and TE polarizations. It appears that in almost all considered cases (except for ϕ = 45°, θ = 0°), the cross-polar coefficients are as large as the co-polar ones around the resonant frequency, as can be seen in Fig 6 (b) for the particular case (TM, ϕ = 0°, θ = 0°), which corresponds to normal incidence with electric field along *x*. These cross-polarization effects are probably a manifestation of magneto-electric coupling. Indeed, the 2D array of SRR does not present enough symmetry to avoid bianosotropy [8]. In other words, this structure cannot be described by the simplified dyadic permittivity and permeability shown in Eq. (3). Therefore, the retrieval procedure developed in Section 3 cannot be applied to this system.

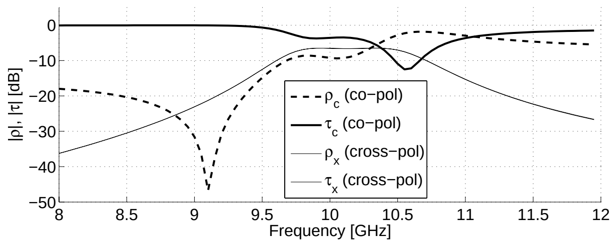
4.4. 2D array of crossed-SRR (CSRR)

Crossed-SRR (CSRR) are volumetric magnetic resonators which exhibit higher level of symmetry than conventional planar SRR [9]. They are made of two or three intersecting SRR, as illustrated in Fig. 7.

Fig. 7 (c) shows the reflection and transmission coefficients for normal incidence with different polarization (different values of ϕ) for the CSRR of Fig. 7 (a). Due to higher level of symmetry compared to the array of SRR of Fig. 6, the cross-polar coefficients are negligible and the co-polar coefficients are ϕ - independent. It can be noted that at the resonance of 0.88 GHz, we have λ/a = 6.8.

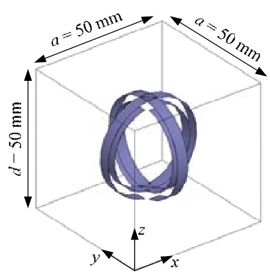


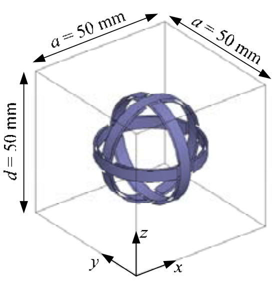
(a)



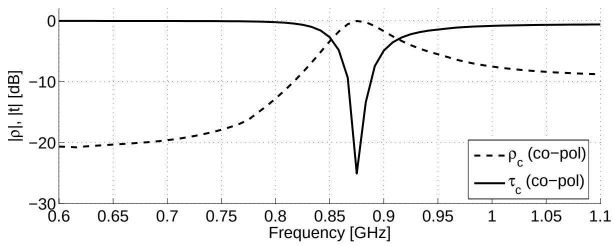
(b)

Figure 6. 2D array of SRR: (a) unit cell, (b) scattering parameters for normal incidence with electric field along x (case TM, ϕ = 0°, θ = 0°). A resonance can be observed around 10.5 GHz. At this frequency, λ/a = 5.7.





(a) (b)



(c)

Figure 7. 2D array of CSRR: (a) and (b) unit cell. The diameter of the CSRR itself is 35 mm, (c) co-polar scattering parameters for normal incidence with different polarizations (cases TM, ϕ = 0°, 15°, 30°, 45° , θ = 0°).

However, this structure has not been investigated further with our retrieval procedure, since it is not symmetric with respect to the *z* axis and is thus not compatible with the model of an anisotropic homogeneous slab. For this reason, the next step will be to investigate the highly symmetrical CSRR presented in Fig. 7 (b). Such magnetic resonators have been reported in [8, 10]. Compared to the CSRR of Fig. 7, this structure contains three intersecting SRR. In order not to destroy their individual resonance, an additional gap has to be considered in each ring. This has the undesirable effect of lowering the total capacitance in the SRR, and thus increasing the resonance frequency, which is why the space between the two rings of each SRR has been reduced to 1mm (in order to increase the capacitance), which renders the numerical analysis rather difficult (difficulty in convergence, prohibitive time) with a FEM solver like HFSS. However, preliminary simulation tests revealed a resonance around 1.28 GHz. At this frequency we have λ/a = 4.7. It is explained in [8] that this structure is isotropic and that it does not exhibit magneto-electric coupling, provided the fact that the unit cell is small enough compared to the wavelength. This remains to be tested with the retrieval procedure developed in Section 3.

5. CONCLUSION

In this work, a retrieval procedure, which allows the determination of equivalent dyadic permittivity and permeability of metamaterials from reflection and transmission coefficients obtained for several incidence directions and polarizations, has been developed. Due to the infinite number of incidence directions that can be considered, this technique offers a redundancy of information and thus several instances of each parameter are obtained, which should be the same if the MTM under study can be described by an homogeneous anisotropic medium. Therefore, this technique can be used to evaluate the accuracy of the homogeneous anisotropic model for specific MTM. Preliminary tests on loaded and unloaded wire media showed that this kind of structures can be accurately described by this model, provided the fact that the unit cell is much smaller than the wavelength (20 times smaller, for the inductively loaded wire medium). The calculation of the reflection and transmission coefficients for arrays of magnetic resonators showed that a certain level of symmetry in the unit cell is required in order to avoid bianisotropy. Further work concerns the investigation of highly symmetrical magnetic resonators (CSRR) with the present retrieval procedure. In particular, their isotropy when they are arranged in arrays is being analysed.

ACKNOWLEDGEMENTS

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Table 1. Dispersion relations and extraction of dyadic permittivity and permeability.

ϕ	Pol.	Dispersion relation	$\tilde{z}$	1st parameter	Relation between 2nd and 3rd parameters
0°	TM	$\frac{\gamma_{2iz}^2}{\varepsilon_x} + \frac{\gamma_{2ix}^2}{\varepsilon_z} + \omega^2 \mu_y = 0$	$\frac{\tilde{n}}{\varepsilon_{rx}}$	$\varepsilon_{rx} = \frac{\tilde{n}}{\tilde{z}}$	$\varepsilon_{rz}\left(\mu_{ry} - \tilde{n}\tilde{z}\cos^2\theta\right) = \sin^2\theta$
90°	TM	$\frac{\gamma_{2iz}^2}{\varepsilon_y} + \frac{\gamma_{2iy}^2}{\varepsilon_z} + \omega^2 \mu_x = 0$	$\frac{\tilde{n}}{\varepsilon_{ry}}$	$\varepsilon_{ry} = \frac{\tilde{n}}{\tilde{z}}$	$\varepsilon_{rz}\left(\mu_{rx} - \tilde{n}\tilde{z}\cos^2\theta\right) = \sin^2\theta$
0°	TE	$\frac{\gamma_{2iz}^2}{\mu_x} + \frac{\gamma_{2ix}^2}{\mu_z} + \omega^2 \varepsilon_y = 0$	$\frac{\mu_{rx}}{\tilde{n}}$	$\mu_{rx} = \tilde{n}\tilde{z}$	$\mu_{rz}\left(\varepsilon_{ry} - \frac{\tilde{n}}{\tilde{z}}\cos^2\theta\right) = \sin^2\theta$
90°	TE	$\frac{\gamma_{2iz}^2}{\mu_y} + \frac{\gamma_{2iy}^2}{\mu_z} + \omega^2 \varepsilon_x = 0$	$\frac{\mu_{ry}}{\tilde{n}}$	$\mu_{ry} = \tilde{n}\tilde{z}$	$\mu_{rz}\left(\varepsilon_{rx} - \frac{\tilde{n}}{\tilde{z}}\cos^2\theta\right) = \sin^2\theta$